Course Title: Advanced Placement Calculus (BC) – Period 7

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Textbook: CALCULUS: Graphical, Numerical, Algebraic
Authored by: Finney, Demana, Waits, and Kennedy
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Course Objectives

* To expose students to the first two semesters of college calculus;

* To prepare students for advanced coursework in mathematics, science, technology, engineering, etc;

* To prepare students for the Advanced Placement Exam in May;

* To provide opportunities for students to integrate the use of modern technologies along with traditional methods of analysis in problem solving and discovery;

* To provide opportunities for students to work cooperatively in problem solving and discovery.

Course Goals

* Students should be able to work with functions represented in several ways: graphically, numerically, analytically, and/or verbally. They should also understand the, and be able to make, connections among these representations;

* Students should understand the meaning of the derivative as a rate of change and in terms of local linear approximations; and should be able to use derivatives to solve problems;

* Students should understand the meaning of the definite integral both as a limit of Reimann sums and as the net accumulation of change; and should be able to use integrals to solve problems;

* Students should understand the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus;

* Students should be able to communicate mathematics both orally and in well written sentences, and should be able to explain solutions to problems;

Course Goals (continued)

* Students should be able to model a written description of a physical situation with a function, a differential equation, or an integral;

* Students should be able to use technology to: help solve problems, explore situations, experiment with ideas, investigate phenomena, interpret results, and verify conclusions;

* Students should be able to determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurement;

* Students should develop an appreciation of calculus as a meaningful and coherent body of knowledge and as a human accomplishment.

Topical Outline

I. Functions, Graphs, and Limits

Analysis of graphs. With the aid of technology, graphs of functions are often easy to produce. Hence the emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

Limits of a function. An intuitive understanding of the limiting process is sufficient for this course.

* Calculating limits using algebra.
* Estimating limits from graphs or tables of data.
  (If time permits we will investigate the formal definition of limits)

Asymptotic and unbounded behavior.

* Understanding asymptotes in terms of graphical behavior.
* Describing asymptotic behavior in terms of limits involving infinity.
* Comparing relative magnitudes of functions and their rates of change.

Continuity as a property of functions. The central idea of continuity is that close values of the domain lead to close values of the range.

* Understanding continuity in terms of limits.
* Geometric understanding of graphs of continuous functions (IVT and EVT).

II. Derivatives

Concept of a derivative. The concept of the derivative is presented geometrically, numerically, and analytically, and is interpreted as an instantaneous rate of change.

* Derivative defined as a limit of the difference quotient.
* Relationship between differentiability and continuity.

Derivative at a point.

* Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
* Tangent line to a curve at a point and local linear approximation.
* Instantaneous rate of change as the limit of average rate of change.
* Approximate rate of change from graphs and tables of values.

Derivative of a function.

* Corresponding characteristics of graphs of **f** and **f'**.
* Relationship between the increasing /decreasing behavior of **f** and the sign of **f'**.
* The Mean Value Theorem (MVT) and its geometric consequences.
* Equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

Second derivatives.

* Corresponding characteristics of the graphs of **f**, **f'**, and **f"**.
* Relationship between the concavity of f and the sign of **f"**.
* Points of inflection as places where the concavity changes.

Applications of derivatives.

* Analysis of curves, including the notions of monotonicity and concavity.
* Optimization, both absolute (global) and relative (local) extrema.
* Modeling rates of change, including related rates problems.
* Use of implicit differentiation to determine the derivative of an inverse function.
* Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration.

Computation of derivatives.

* Knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions.
* Basic rules for the derivative of sums, products, and quotients of functions.
* Chain rule and implicit differentiation.

III. Integrals

Reimann sums.

* Concept of a Reimann sum over equal subdivisions.
* Computation of Reimann sums using left, right, and midpoint evaluation points.

Interpretations and properties of definite integrals.

* Definite integral as a limit of Reimann sums.
* Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:

$$f'(x)dx = f(b) - f(a).$$

* Basic properties of definite integrals.

Applications of integrals. Appropriate integrals are used in a variety of applications to model physical, social, or economic situations. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the integral of a rate of change to yield accumulated change or using the method of setting up an approximating Reimann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include finding the area of a region, the volume of a solid with known cross section, the average value of a function, and the distance traveled by a particle along a line.

Fundamental Theorem of Calculus.

* Use of the Fundamental Theorem to evaluate definite integrals.
* Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined.

Techniques of antidifferentiation.

* Antiderivatives following directly from derivatives of basic functions.
* Antiderivatives by substitution of variables.

Applications of antidifferentiation.

* Determining specific antiderivatives using initial conditions.
* Solving separable differential equations and using them in modeling.

Numerical approximation to definite integrals. Use of Reimann sums and the Trapezoidal rule to approximate definite integrals of functions represented algebraically, geometrically, and by tables of values.

IV. Topics/Techniques in BC Calculus (not covered in AB Calculus)

- Parametric Functions
- Vector Functions
- Polar Curves
- Euler's Method for approximating numerical solutions to differential equations
- L'Hopital's Rule for determining limits involving indeterminate forms
- Length of a Curve
- Integration by Parts
- Integration by Partial Fractions
- Evaluating Improper Integrals
- Solving Logistic Differential Equations
- Series
  - Types of Series (geometric, power, p-series, alternating, harmonic, etc.)
  - Functions defined as a Series
  - Calculus of Series - Differentiating and Integrating Series
  - Evaluating Series - Convergence/Divergence
  - Radius/Integral of Convergence
  - Tests for Convergence/Divergence
  - Comparing Series
  - Taylor/Maclaurin Polynomials for Approximating Series
  - Error Investigation
    - Error Evaluation
    - Error Bounds